

Math 115

Spring 2017

Lecture 24

① Simplify: $\frac{x^3 - 5x^2 + 6x}{x^4 - 4x^2} = \frac{x(x^2 - 5x + 6)}{x^2(x^2 - 4)}$

$= \frac{\cancel{x}(x-2)(x-3)}{x^{\cancel{2}}(x+2)(x-2)} = \boxed{\frac{x-3}{x(x+2)}}$

② Find all the excluded values: $\frac{x^2 - 10}{3x^2 - x - 4}$

Restricted Values \rightarrow Deno = 0, Solve

$3x^2 \boxed{-x} - 4 = 0$
 $\swarrow \quad \uparrow$
 $-12 \quad P = -12, S = -1$
 $3 \nmid -4$

$3x^2 \boxed{+3x - 4x} - 4 = 0$
 $3x(x+1) - 4(x+1) = 0$
 $(x+1)(3x-4) = 0$

\rightarrow by Z.F.P.
 $x+1=0$ or $3x-4=0$
 $\boxed{x=-1}$ $\boxed{x=\frac{4}{3}}$
 E.V.: $-1, 4/3$

Multiply

$$\frac{x^2 + 6x}{x^2 + 6x + 9} \cdot \frac{x^2 - 9}{x^2 + 3x - 18}$$

$$= \frac{\cancel{x}(x+6)}{\cancel{(x+3)}(x+3)} \cdot \frac{\cancel{(x+3)}\cancel{(x-3)}}{\cancel{(x+6)}(x-3)} = \boxed{\frac{x}{x+3}}$$

Divide:

$$\frac{3x^2 - 13x + 4}{9x^2 - 1} \div \frac{x^2 - 16}{3x^2 + x}$$

$$= \frac{\cancel{(x-4)}\cancel{(3x-1)}}{\cancel{(3x+1)}\cancel{(3x-1)}} \cdot \frac{x\cancel{(3x+1)}}{\cancel{(x+4)}\cancel{(x-4)}} = \boxed{\frac{x}{x+4}}$$

$$\begin{aligned} & 3x^2 - 13x + 4 \\ & \quad \swarrow \quad \searrow \quad P=2 \quad S=-4 \\ & \quad 12 \quad S=-4 \\ & \quad -12 \quad \cancel{+1} \\ & \quad = 3x^2 - 12x - 1x + 4 \\ & \quad = 3x(x-4) - 1(x-4) \end{aligned}$$

Adding / Subtracting like fractions

Subtract:

$$\frac{2x}{x^2 - 25} - \frac{10}{x^2 - 25}$$

$$= \frac{2x - 10}{x^2 - 25}$$

$$= \frac{2\cancel{(x-5)}}{(x+5)\cancel{(x-5)}} = \boxed{\frac{2}{x+5}}$$

Same Denominator

$$\begin{aligned} & \frac{A}{D} + \frac{B}{D} - \frac{C}{D} \\ & = \frac{A + B - C}{D} \end{aligned}$$

then reduce

Simplify: $\frac{x^2 - 2x + 3}{x^2 - 4} + \frac{x + 3}{x^2 - 4}$

$$= \frac{x^2 - 2x + 3 + x + 3}{x^2 - 4} = \frac{x^2 - x + 6}{x^2 - 4}$$

Can not be reduced.

Simplify:

$\frac{x^2 + 9x}{x + 7} - \frac{4x + 14}{x + 7}$

$$= \frac{x^2 + 9x - 4x - 14}{x + 7} = \frac{x^2 + 5x - 14}{x + 7} = \frac{(x + 7)(x - 2)}{x + 7} = \boxed{x - 2}$$

Simplify: $\frac{3x - 1}{x^2 + 5x - 6} - \frac{2x - 7}{x^2 + 5x - 6}$

$$= \frac{3x - 1 - (2x - 7)}{x^2 + 5x - 6}$$

$$= \frac{3x - 1 - 2x + 7}{x^2 + 5x - 6} = \frac{x + 6}{x^2 + 5x - 6}$$

$$= \frac{1(x + 6)}{(x + 6)(x - 1)}$$

$$= \boxed{\frac{1}{x - 1}}$$

Now unlike fractions

$$\begin{aligned}
 & \frac{5}{x^2 - 4} + \frac{2}{x^2 + 5x + 6} \\
 &= \frac{5(x+3)}{(x+2)(x-2)(x+3)} + \frac{2(x-2)}{(x+3)(x+2)(x-2)} \\
 &= \frac{5(x+3) + 2(x-2)}{(x+2)(x-2)(x+3)} = \boxed{\frac{7x + 11}{(x+2)(x-2)(x+3)}}
 \end{aligned}$$

Simplify

$$\begin{aligned}
 & \frac{x}{x^2 - 16} - \frac{4}{x^2 - 6x + 8} = \\
 & \frac{x(x-2)}{(x+4)(x-4)(x-2)} - \frac{4(x+4)}{(x-2)(x-4)(x+4)} \\
 &= \frac{x(x-2) - 4(x+4)}{(x+4)(x-4)(x-2)} = \frac{x^2 - 2x - 4x - 16}{(x+4)(x-4)(x-2)} \\
 &= \boxed{\frac{x^2 - 6x - 16}{(x+4)(x-4)(x-2)}} = \boxed{\frac{(x+2)(x-8)}{(x+4)(x-4)(x-2)}}
 \end{aligned}$$

Simplify:

$$\begin{aligned}
 & \frac{x}{x^2-4} - \frac{5}{x^2-4x+4} \\
 &= \frac{x(x-2)}{(x-2)(x+2)(x-2)} - \frac{5(x+2)}{(x-2)(x-2)(x+2)} \\
 &= \frac{x^2-2x-5x-10}{\underbrace{(x-2)(x+2)(x-2)}_{\text{LCD}}} = \frac{x^2-7x-10}{(x+2)(x-2)^2}
 \end{aligned}$$

Solving Rational Equations

$$\frac{3}{x-5} - \frac{2}{x+5} = \frac{30}{x^2-25}$$

$$\text{LCD} = (x-5)(x+5) \rightarrow 5 \text{ is an E.V.}$$

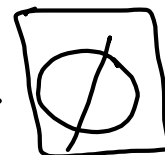
$$\cancel{(x-5)}(x+5) \cdot \frac{3}{\cancel{x-5}} - \cancel{(x-5)}(x+5) \cdot \frac{2}{\cancel{x+5}} =$$

$$3(x+5) - 2(x-5) = 30$$

$$3x + 15 - 2x + 10 = 30$$

$$x + 25 = 30 \rightarrow$$

$$\boxed{x=5}$$

 \Rightarrow


Since 5
is an E.V.